

KARIMPUR PANNADEVI COLLEGE
MATHEMATICS-HONOURS (Second Semester)

Internal Assessment, 2020

Duration: 45 mins.

Full Marks: 20

Group-A (MATH-H-CC-T-03)

Answer any two questions

Q.1) Prove that the set \mathbb{R} of all real numbers is dense. If a and b are two positive real numbers then show that there exists a positive integer n such that $na > b$. 2+3 marks

Q.2) Prove that arbitrary intersection of closed sets is closed. Give an example to show that arbitrary union of closed sets may not be closed. 3+2 marks

Q.3) Prove that $K \subset \mathbb{R}$ is closed if and only if any convergent sequence of elements in K , the limit of the sequence is in K . From the completeness property in \mathbb{R} , Prove that $\lim \frac{1}{n} = 0$. 3+2 marks

Q.4) If $x_n = \left(1 + \frac{1}{n}\right)^n$, prove that the sequence $\{x_n\}_n$ is bounded. If $\{a_n\}_n$ is a convergent sequence of positive terms and $a_{n+1} = \sqrt{2a_n}$. Find $\lim a_n$. 2+3 marks

Q.5) Prove Bolzano Weierstrass' theorem from Heine Borel's theorem. 5 marks

Group-B (MATH-H-CC-T-04)

Answer any two questions

Q.6) Solve the differential equation: $\frac{d^2y}{dx^2} + a^2y = \tan ax$ 5 marks

Q.7) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = \frac{e^{-x}}{x^2}$ 5 marks

Q.8) Solve: $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x}\right)$. 5 marks

Q.9) Find the eigenvalues and the eigenfunctions of the B.V.P. $\frac{d^2y}{dx^2} + \lambda y = 0$ ($\lambda > 0$) under the boundary conditions $y(0) = 0$ and $y'(0) = \pi$. 5 marks

Q.10) Show that a necessary and sufficient condition for a scalar point function ϕ to be constant is that $\vec{\nabla} \phi = \vec{0}$. Find the maximum value of the directional derivative of $\phi = x^2 - y^2 + z^2$ at the point $(1, 3, 2)$. 3+2 marks